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MULTIPLE-DECISION SELECTION AND  
RANKING PROCEDURES

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**MULTIPLE-DECISION SELECTION  
AND RANKING PROCEDURES**

**FINAL REPORT**

by

**Robert E. Bechhofer**

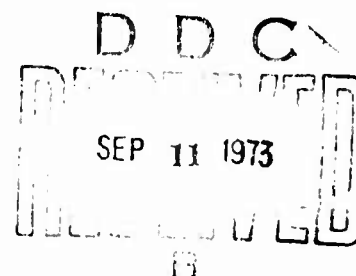
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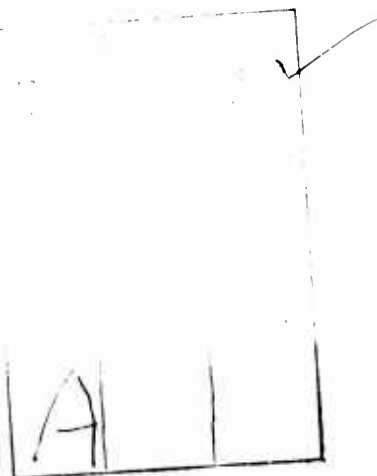
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FINAL REPORT ON CONTRACT DA-31-124-ARO-D-474

This is the final report on Contract DA-31-124-ARO-D-474 entitled "Multiple-Decision Selection and Ranking Procedures." The initial contract covered a three-year period starting September 15, 1966; the contract was renewed for a second three-year period starting September 15, 1969; a no-cost extension was given until January 31, 1973 and a second such extension until June 30, 1973.

The following pages list the technical reports and published papers (and those accepted for or submitted for publication) which were written with the full or partial support of the contract; the contents of these reports and papers have already been described in the semi-annual progress reports. Also included is a list of the scientific personnel supported by the contract, and the advanced degrees earned by them while supported by the contract.

Most of the papers deal with research on the subject of "Statistical Multiple Decision Ranking Procedures," a statistical methodology pioneered by the Principal Investigator and several colleagues; some of the papers deal with related statistical techniques, and a few with new probabilistic developments.

Some of the many results obtained on this contract were reported on at the meetings of professional societies. Past results and future plans were outlined at the Chief Investigators Conference and Review of the Military Theme "OPERATIONS RESEARCH" held at Aberdeen Proving Ground, July 18-19, 1972. The Principal Investigator has been a regular participant in the annual Design of Experiments Conferences sponsored by ARO-D.

The mission of this contract is being continued under a new contract, ARMY DAKC64-73-C-0008. The Principal Investigator is deeply indebted to the Army Research Office-Durham for its strong support of this activity.

Summary of Research Completed on the Contract

Technical reports, the research for which was supported in whole or in part by ARO Contract DA-31-124-ARO-D-474.

Bechhofer, Robert E.: Single-stage procedures for ranking multiply-classified variances of normal populations, Technical report No. 41 dated February 1968.

Bechhofer, Robert E.: Multiple comparisons with a control for multiply-classified variances of normal populations, Technical report No. 42 dated February 1968.

Higgins, James E.: Bernoulli sampling plans which approximately minimize the maximum expected sample size subject to certain probability requirements, Technical report No. 65 dated February 1969.

Ramberg, John S.: A multiple-decision approach to the selection of the best set of predictor variates, Technical report No. 79 dated July 1969.

Bawa, Vijay S.: Asymptotically optimal ranking and selection procedures, Technical report No. 102 dated March 1970.

Nocturne, D.J.M.: Asymptotic efficiency of the maximum likelihood estimators for the parameters of certain stochastic processes, Technical report No. 105 dated March 1970.

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Fushimi, Masanori: An improved version of a Sobel-Weiss play-the-winner procedure for selecting the better of two binomial populations, Technical report No. 170 dated January 1973.

Frischtak, Ricardo: Statistical multiple-decision procedures for some multivariate selection problems, Technical report No. 187 dated July 1973.

Fushimi, Masanori: A non-symmetric sequential procedure for selecting the better of two binomial populations, Technical report No. 189 dated August 1973.

Papers (published), the research for which was supported in whole or in part by ARO contract DA-31-124-ARO-D-474.

Bawa, V.S.: "Asymptotic efficiency of one R-factor experiment relative to R one-factor experiments for selecting the best normal population," Journal of the American Statistical Association, Vol. 67 (1972), pp. 660-661.

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Turnbull, B.W.: "Inequalities for multitype branching processes," Annals of Probability, Vol. 1, No. 3 (1973), pp. 475-479.

Paper (accepted for publication), the research for which was supported in whole or in part by ARO contract DA-31-124-ARO-D-474.

Fushimi, M.: "An improved version of a Sobel-Weiss play-the-winner procedure for selecting the better of two binomial populations." Accepted for publication in Biometrika.

Papers (submitted for publication), the research for which was supported in whole or in part by ARO contract DA-31-124-ARO-D-474.

Fushimi, M.: "A non-symmetric sequential procedure for selecting the better of two binomial populations." Submitted for publication to Biometrika.

Nocturne, D.: "Asymptotic efficiency of the maximum likelihood estimator for the parameters of certain stochastic processes." Submitted for publication to the Annals of Statistics.

Graduate students supported by ARO contract DA-31-124-ARO-D-474.

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Bawa, Vijay S.: (Ph.D. dissertation) "Asymptotically optimal ranking and selection procedures." June 1970.

Frischtak, Ricardo M.: (Ph.D. dissertation) "Statistical multiple-decision procedures for some multivariate selection problems." August 1973.

Higgins, James E.: (M.S. thesis) "Bernoulli sampling plans which approximately minimize the maximum expected sample size." February 1969.

Hooke, John: (Ph.D. dissertation) "Some limit theorems for priority queues." January 1970.

Kakumanu, Prasad Rao V.: (Ph.D. dissertation) "Continuous time Markov decision models with applications to optimization problems." September 1969.

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Ramberg, John S.: (Ph.D. dissertation) "A multiple-decision approach to the selection of the best set of predictor variates." February 1969.

Resnick, Mrs. Yvonne (Kramer): (M.S. thesis) "Hedging procedure for estimation of  $(k+1)$  quantiles for a population with unknown mean." September 1969.

Turnbull, Bruce W.: (Ph.D. dissertation) "Bounds and optimal strategies for stochastic systems." September 1971.